

Exam P

Flashcards

Spring 2009 exams

Key concepts

Important formulas

Efficient methods

*Advice on exam
technique*

CONTENTS

Contents	page
How to use these flashcards	2
Probability	3
Counting techniques	6
Conditional probability	8
Bayes' Theorem	10
Discrete random variables	11
Common discrete distributions	14
Continuous random variables	18
Common continuous distributions	21
The normal distribution	25
Bivariate distributions	30
Conditional expectation and variance	36
The bivariate normal distribution	37
Transformations of random variables	38
Moment generating functions	44
Insurance concepts and terminology	47

HOW TO USE THESE FLASHCARDS

These flashcards are designed to help you to prepare efficiently in the run-up to the Course P exam of the Society of Actuaries. They include conceptual ideas, key formulas and techniques for efficient problem solving. Typical questions on a Course P examination require students to understand and apply several concepts in order to set up a solution, and then perform a series of computations to complete it. So don't look at the lists of formulas as simply being memorization work. There are often simple ideas that underlie the formulas as well as basic mathematical reasons why they are correct. Strive to understand and learn the key relations from this point of view and your knowledge will not be the superficial type that may collapse under the stress of taking the examination. The more that you understand conceptually, the easier it becomes to retain the key ideas and write them down quickly and accurately. Your understanding of probability concepts plays a huge role in Courses M and C that lie ahead where you will encounter more advanced and somewhat abstract probability concepts. You will need a solid foundation to be successful there.

We have designed the flashcards so that they can be carried conveniently and read frequently in the final run-up to the exam, *eg* when sitting on a plane. We hope that you will personalize them by adding your own comments and notes, and checking each section when you feel confident with the material covered.

You will probably also find these summaries useful when you are at the stage of working through the past exams. The BPP Exam P Question and Answer Bank contains a mixture of past exam questions and brand new exam-style questions, along with detailed solutions. By the time you have worked through these questions you will have a clear picture of what the exam is like and what you need to work on to get ready for it. As a final tune-up, try one of the BPP Course P practice exams, containing all new exam-style questions.

Good luck with your studying.

PROBABILITY

Set theory

The *complement* of subset A is written as A' or \bar{A} , and is the subset of all elements that are not elements of A .

The *union* of subsets A and B , which is written as $A \cup B$, is the subset of all elements that are elements of either A or B , or both.

The *intersection* of subsets A and B , which is written as $A \cap B$, is the subset of all elements that are elements of both A and B .

The empty set (a set with no elements) is denoted \emptyset .

Sets A_1, A_2, \dots are *mutually exclusive* if $A_i \cap A_j = \emptyset$ whenever $i \neq j$. The sets are *exhaustive* if their union is S , the entire sample space. If the sets are both mutually exclusive and exhaustive, then they form a *partition* of the sample space.

Basic laws of set theory

1. The *associative laws* state that:

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

2. The *distributive laws* state that:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

3. *De Morgan's laws* state that:

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

4. If events A_1, A_2, \dots are mutually exclusive and exhaustive, then:

$$B = (B \cap A_1) \cup (B \cap A_2) \cup \dots$$

Hint: Draw Venn diagrams to help understand these laws.

CONDITIONAL PROBABILITY

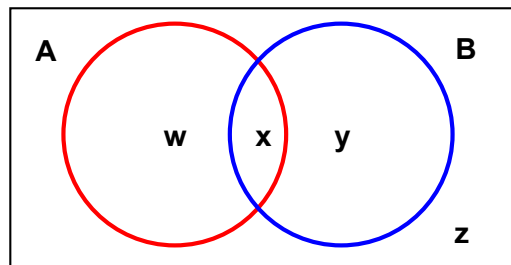
Conditional probability

The *conditional probability of A given B* is denoted $\Pr(A|B)$.

The conditional probability is defined to be:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} \quad \text{where } \Pr(B) > 0$$

If we represent events A and B using a Venn diagram:



then the main relationships are as follows:

$$\Pr(A) = w + x$$

$$\Pr(B) = x + y$$

$$\Pr(A \cap B) = x$$

$$\Pr(A \cup B) = w + x + y = 1 - z$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{x}{x + y}$$

$$\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)} = \frac{x}{w + x}$$

THE NORMAL DISTRIBUTION

The normal distribution

The normal distribution is perhaps the most important continuous random variable. The pdf of the normal distribution has two parameters, μ and σ^2 , that are the mean and variance.

Probability density function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \quad \text{for } -\infty < x < \infty$$

Expectation:

$$E[X] = \mu$$

Variance:

$$\text{var}(X) = \sigma^2$$

The notation $X \sim N(\mu, \sigma^2)$ means that the random variable X is normally distributed with mean μ and variance σ^2 .

The pdf has a symmetric, bell-shaped graph centered at μ .

A normal distribution with a greater standard deviation has a lower and flatter pdf, because there is a greater chance of observing values far away from the mean. A normal distribution with a lower standard deviation has a higher and narrower pdf, because there is less chance of observing values far away from the mean.

