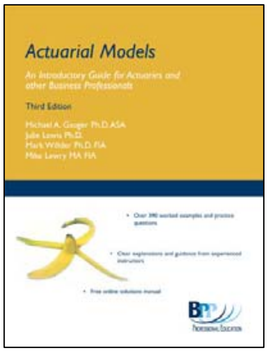

Actuarial Models

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By M Gauger, J Lewis, M Willder, M Lewry

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Solutions to practice questions – Chapter 5

Solution 5.1

The benefit reserve at duration t is a measure of liability for a policy issued at age x that is still in force at age $x+t$. It is equal to the APV at age $x+t$ of the future benefits less the APV at age $x+t$ of the future premiums.

Solution 5.2

The reserve can be computed retrospectively by thinking of the asset the insurer holds that matches the prospective liability. Think of the spreadsheet used to compute aggregate deterministic results. The asset is the ending fund at the end of year t allocated to the l_{x+t} survivors. So the retrospective reserve is a survivor's share of the accumulation of past premium less the accumulation of past benefits.

Solution 5.3

(i) $$1{,}000\,{}_5V\left(\bar{A}_{40}\right) = 1{,}000\,\bar{A}_{45} - 1{,}000\,P\left(\bar{A}_{40}\right)\ddot{a}_{45} = \frac{1{,}000\,P\left(\bar{A}_{40}\right)\ddot{a}_{40:\overline{5}|}}{v^5\,{}_5p_{40}} - \frac{1{,}000\,\bar{A}^{\;1}_{40:\overline{5}|}}{v^5\,{}_5p_{40}}$$

(ii) $$1{,}000\,{}_5\bar{V}\left(\bar{A}^{1}_{40:\overline{10}|}\right) = 1{,}000\,\bar{A}^{1}_{45:\overline{5}|} - 1{,}000\,\bar{P}\left(\bar{A}^{1}_{40:\overline{10}|}\right)\bar{a}_{45:\overline{5}|} = \frac{1{,}000\,\bar{P}\left(\bar{A}^{1}_{40:\overline{10}|}\right)\bar{a}_{40:\overline{5}|}}{v^5\,{}_5p_{40}} - \frac{1{,}000\,\bar{A}^{1}_{40:\overline{5}|}}{v^5\,{}_5p_{40}}$$

(iii) $$1{,}000\,{}_5V_{40:\overline{10}|} = 1{,}000\,A_{45:\overline{5}|} - 1{,}000\,P_{40:\overline{10}|}\,\ddot{a}_{45:\overline{5}|} = \frac{1{,}000\,P_{40:\overline{10}|}\,\ddot{a}_{40:\overline{5}|}}{v^5\,{}_5p_{40}} - \frac{1{,}000\,A^{\;1}_{40:\overline{5}|}}{v^5\,{}_5p_{40}}$$

Solution 5.4

(i) The insurance is a 20-payment, semi-continuous whole life insurance of 1,000 issued on a life age 40. The policy is still in force 10 years after issue.

(ii) The insurance is a 20-payment, fully discrete whole life insurance of 1,000 issued on a life age 40 with premiums paid in semi-annual installments. The policy is still in force 10 years after issue.

Solution 5.5

(i) $$1{,}000\ {}_{10}^{20}V(\bar{A}_{40}) = 1{,}000\,\bar{A}_{50} - 1{,}000\ {}_{20}P(\bar{A}_{40})\,\ddot{a}_{50:\overline{10|}} = \frac{1{,}000\ {}_{20}P(\bar{A}_{40})\,\ddot{a}_{40:\overline{10|}}}{v^{10}\ {}_{10}p_{40}} - \frac{1{,}000\,\bar{A}^{1}_{40:\overline{10|}}}{v^{10}\ {}_{10}p_{40}}$$

(ii) $$1{,}000\ {}_{10}^{20}V_{40}^{(2)} = 1{,}000\left(A_{50} - {}_{20}P_{40}^{(2)}\ \ddot{a}^{(2)}_{50:\overline{10|}}\right) = 1{,}000\left(\frac{{}_{20}P_{40}^{(2)}\ \ddot{a}^{(2)}_{40:\overline{10|}}}{v^{10}\ {}_{10}p_{40}} - \frac{A^{1}_{40:\overline{10|}}}{v^{10}\ {}_{10}p_{40}}\right)$$

Solution 5.6

(i) $$1{,}000\ {}_{30}^{20}V(\bar{A}_{40}) = 1{,}000\,\bar{A}_{70}\ \text{(no future premium)}$$

(ii) $$1{,}000\ {}_{30}^{20}V_{40}^{(2)} = 1{,}000\,A_{70}\ \text{(no future premium)}$$

Solution 5.7

$$1{,}000\,P\left(\bar{A}_{40}\right) = \frac{1{,}000\,\bar{A}_{40}}{\ddot{a}_{40}} = \frac{i}{\delta}\times\frac{1{,}000\,A_{40}}{\ddot{a}_{40}} = \frac{i}{\delta}\times 1{,}000\,P_{40}$$

$$1{,}000\ {}_5V\left(\bar{A}_{40}\right) = 1{,}000\,\bar{A}_{45} - 1{,}000\,P\left(\bar{A}_{40}\right)\ddot{a}_{45}$$

$$= \frac{i}{\delta}\times 1{,}000\,A_{45} - \frac{i}{\delta}\times 1{,}000\,P_{40}\,\ddot{a}_{45}$$

$$= \frac{i}{\delta}\times 1{,}000\ {}_5V_{40} = \frac{i}{\delta}\times 1{,}000\left(1-\frac{\ddot{a}_{45}}{\ddot{a}_{40}}\right)$$

$$= 1{,}029.71\left(1-\frac{14.1121}{14.8166}\right) = 48.96067$$

Solution 5.8

$$A^{1}_{x:\overline{3|}} = \frac{0.02}{1.05} + \frac{0.98\times 0.03}{1.05^2} + \frac{0.98\times 0.97\times 0.04}{1.05^3} = 0.07856$$

$$A^{1}_{x+1:\overline{2|}} = \frac{0.03}{1.05} + \frac{0.97\times 0.04}{1.05^2} = 0.06376$$

$$A^{1}_{x+2:\overline{1|}} = \frac{0.04}{1.05} = 0.03810$$

$$\ddot{a}_{x:\overline{3|}} = 1 + \frac{0.98}{1.05} + \frac{0.98\times 0.97}{1.05^2} = 2.79556 \ \Rightarrow\ P^{1}_{x:\overline{3|}} = \frac{A^{1}_{x:\overline{3|}}}{\ddot{a}_{x:\overline{3|}}} = 0.02810$$

$$\ddot{a}_{x+1:\overline{2|}} = 1 + \frac{0.97}{1.05} = 1.92381\ ,\quad \ddot{a}_{x+2:\overline{1|}} = 1$$

$${}_1V^{1}_{x:\overline{3|}} = A^{1}_{x+1:\overline{2|}} - P^{1}_{x:\overline{3|}}\,\ddot{a}_{x+1:\overline{2|}} = 0.00970$$

$${}_2V^{1}_{x:\overline{3|}} = A^{1}_{x+2:\overline{1|}} - P^{1}_{x:\overline{3|}}\,\ddot{a}_{x+2:\overline{1|}} = 0.00999$$

Solution 5.9

$$\ddot{a}_{x:\overline{2|}} = 1 + \frac{0.98}{1.05} = 1.93333 \ , \ A^{1}_{x:\overline{2|}} = \frac{0.02}{1.05} + \frac{0.98 \times 0.03}{1.05^2} = 0.04571$$

$$_2V^{1}_{x:\overline{3|}} = \frac{P^{1}_{x:\overline{3|}}\ \ddot{a}_{x:\overline{2|}}}{v^2\ _2p_x} - \frac{A^{1}_{x:\overline{2|}}}{v^2\ _2p_x} = \frac{0.02810 \times 1.93333}{1.05^{-2} \times 0.98 \times 0.97} - \frac{0.04571}{1.05^{-2} \times 0.98 \times 0.97} = 0.00999$$

Solution 5.10

$$_0V^{1}_{x:\overline{3|}} + P^{1}_{x:\overline{3|}} = v\,q_x + v\,p_x\ _1V^{1}_{x:\overline{3|}} \ \Rightarrow \ _1V^{1}_{x:\overline{3|}} = \frac{_0V^{1}_{x:\overline{3|}} + P^{1}_{x:\overline{3|}} - v\,q_x}{vp_x} = \frac{0 + 0.02810 - 0.02/1.05}{0.98/1.05} = 0.00970$$

$$_2V^{1}_{x:\overline{3|}} = \frac{_1V^{1}_{x:\overline{3|}} + P^{1}_{x:\overline{3|}} - v\,q_{x+1}}{vp_{x+1}} = \frac{0.00970 + 0.02810 - 0.03/1.05}{0.97/1.05} = 0.00999$$

Solution 5.11

Event	Loss function value	Probability
$K(x)=0$	$v - P^{1}_{x:\overline{3|}} = 0.92428$	$q_x = 0.02$
$K(x)=1$	$v^2 - P^{1}_{x:\overline{3|}}(1+v) = 0.85216$	$_{1\|}q_x = 0.98 \times 0.03 = 0.02940$
$K(x)=2$	$v^3 - P^{1}_{x:\overline{3|}}\left(1+v+v^2\right) = 0.78348$	$_{2\|}q_x = 0.98 \times 0.97 \times 0.04 = 0.03802$
$K(x)\geq 3$	$0 - P^{1}_{x:\overline{3|}}\left(1+v+v^2\right) = -0.08036$	$_3p_x = 0.98 \times 0.97 \times 0.96 = 0.91258$

Solution 5.12

You can check that the expected loss function value is zero. So the variance is:

$$\begin{aligned}\text{var}\left(_0L\right) = E\left[_0L^2\right] &= (0.92428)^2\,(0.02) + (0.85216)^2\,(0.02940) \\ &\quad + (0.78348)^2\,(0.03802) + (-0.08036)^2\,(0.91258) = 0.06767\end{aligned}$$

Solution 5.13

$$\text{var}\left(_2L\right) = v^2\, q_{x+2}\, p_{x+2}\left(1 - {_3V^{1}_{x:\overline{3|}}}\right)^2 = 0.03483$$

$$\text{var}\left(_1L\right) = v^2\, q_{x+1}\, p_{x+1}\left(1 - {_2V^{1}_{x:\overline{3|}}}\right)^2 + v^2\, p_{x+1}\ \text{var}\left(_2L\right) = 0.05651$$

$$\text{var}\left(_0L\right) = v^2\, q_x\, p_x\left(1 - {_1V^{1}_{x:\overline{3|}}}\right)^2 + v^2\, p_x\ \text{var}\left(_1L\right) = 0.06767$$

This result agrees with the result obtained in Solution 5.12.

Solution 5.14

$$P\,\ddot{a}_{55:\overline{10|}} = 50{,}000\ {}_{10|}\ddot{a}_{55} = 50{,}000\,v^{10}\ {}_{10}p_{55}\,\ddot{a}_{65} \Rightarrow P = \frac{50{,}000\,v^{10}\ {}_{10}p_{55}\,\ddot{a}_{65}}{\ddot{a}_{55} - v^{10}\ {}_{10}p_{55}\,\ddot{a}_{65}}$$

Solution 5.15

$${}_5V = 50{,}000\ {}_{5|}\ddot{a}_{60} - P\,\ddot{a}_{60:\overline{5|}}$$

$${}_{15}V = 50{,}000\,\ddot{a}_{70} \quad (\text{no future premium})$$

Solution 5.16

$${}_5V = P\,\ddot{s}_{55:\overline{5|}} \quad (\text{no past benefit at duration 5})$$

$${}_{15}V = \frac{P\,\ddot{s}_{55:\overline{10|}}}{v^5\ {}_5p_{65}} - \frac{50{,}000\,\ddot{a}_{65:\overline{5|}}}{v^5\ {}_5p_{65}}$$

Solution 5.17

With de Moivre's law there is a quick way to calculate the value of a term insurance:

$$A^1_{x:\overline{n|}} = \frac{a_{\overline{n|}}}{\omega - x}$$

There is quite a list of values we need for the reserve at duration 10:

$$A^1_{40:\overline{20|}} = \frac{a_{\overline{20|}}}{60} = 0.20770 \qquad A_{40:\overline{20|}} = A^1_{40:\overline{20|}} + v^{20}\ {}_{20}p_{40} = 0.45896 \qquad \ddot{a}_{40:\overline{20|}} = \frac{1 - A_{40:\overline{20|}}}{d} = 11.36177$$

$$A^1_{50:\overline{10|}} = \frac{a_{\overline{10|}}}{50} = 0.15443 \qquad A_{50:\overline{10|}} = A^1_{50:\overline{10|}} + v^{10}\ {}_{10}p_{50} = 0.64557 \qquad \ddot{a}_{50:\overline{10|}} = \frac{1 - A_{50:\overline{10|}}}{d} = 7.44313$$

$$\begin{aligned} 1{,}000\ {}_{10}V^1_{40:\overline{20|}} &= 1{,}000\,A^1_{50:\overline{10|}} - 1{,}000\,P^1_{40:\overline{20|}}\,\ddot{a}_{50:\overline{10|}} \\ &= 154.43 - \frac{207.70}{11.36177} \times 7.44313 = 18.36757 \end{aligned}$$

Solution 5.18

$$1{,}000\ {}_9V^1_{40:\overline{20|}} + \underbrace{1{,}000\,P^1_{40:\overline{20|}}}_{18.28091} = \underbrace{v\,q_{49}}_{\frac{1}{1.05\times 51}}\ 1{,}000 + \underbrace{v\,p_{49}}_{\frac{50}{1.05\times 51}}\ \underbrace{1{,}000\ {}_{10}V^1_{40:\overline{20|}}}_{18.36757}$$

$$\Rightarrow 1{,}000\ {}_9V^1_{40:\overline{20|}} = 17.54316$$

Solution 5.19

$$18.28091 = \underbrace{\left(v\,1{,}000\;{}_{10}V^{1}_{40:\overline{20|}} - 1{,}000\;{}_{9}V^{1}_{40:\overline{20|}}\right)}_{\text{savings deposit}} + v\,q_{49}\underbrace{\left(1{,}000 - 1{,}000\;{}_{10}V^{1}_{40:\overline{20|}}\right)}_{\text{amount at risk in year 10}}$$

$$= \left(18.36757/1.05 - 17.54316\right) + 1.05^{-1}\left(1/51\right)\left(1{,}000 - 18.36757\right)$$
$$= -0.05023 + 18.33114$$

Note. This may look strange to have a negative savings fund deposit (a withdrawal). But keep in mind the following. The reserves on this term insurance will increase for a few durations, and then they will begin to decrease until the final reserve at duration 20 reaches zero. The savings fund balance always matches the reserve. So when the reserve starts decreasing, the savings fund deposit each year must be negative.

Solution 5.20

The reserve at duration 9.25 using the exact method is:

$${}_{0.75}p_{49.25} = \frac{l_{50}}{l_{59.25}} = \frac{50}{50.75} \;\Rightarrow\; {}_{0.75}q_{49.25} = 1 - {}_{0.75}p_{49.25} = \frac{0.75}{50.75}$$

$$1{,}000\;{}_{9.25}V^{1}_{40:\overline{20|}} = v^{0.75}\;{}_{0.75}q_{49.25}\;1{,}000 + v^{0.75}\;{}_{0.75}p_{49.25}\;1{,}000\;{}_{10}V^{1}_{40:\overline{20|}}$$
$$= \frac{0.75\times 1{,}000}{1.05^{0.75}\times 50.75} + \frac{50\times 18.36757}{1.05^{0.75}\times 50.75} = 31.69323$$

Using interpolation between terminal reserves plus the unearned premium, we have:

$$1{,}000\;{}_{9.25}V^{1}_{40:\overline{20|}} = \left(1{,}000\;{}_{9}V^{1}_{40:\overline{20|}}\right)(1-0.25) + \left(1{,}000\;{}_{10}V^{1}_{40:\overline{20|}}\right)(0.25) + \left(1{,}000\,P^{1}_{40:\overline{20|}}\right)(1-0.25)$$
$$= 17.54316\times 0.75 + 18.36757\times 0.25 + 18.28091\times 0.75 = 31.45994$$

