SOA Exam M
Life Contingencies
Flashcards

Spring 2009 exams

Key concepts

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HOW TO USE THESE FLASHCARDS

These flashcards are designed to help you to prepare efficiently in the run-up to the Life Contingencies segment (**MLC**) of Course M exam of the Society of Actuaries. They include conceptual ideas, key formulas and techniques for efficient problem solving. **MLC** has a number of problems that require first principles reasoning as well as a fair amount of computation. So don't look at the lists of formulas as simply being memorization work. There are often simple intuitive ideas that underlie the formulas as well as basic mathematical reasons why they are correct. Strive to understand and learn the key relations from these points of view and your knowledge will not be the superficial type that may collapse under the stress of taking the examination. The more that you understand, the easier it becomes to retain the key ideas and write them down quickly and accurately.

We have designed the flashcards so that they can be carried conveniently and read frequently in the final run-up to the exam, *eg* when commuting to work. We hope that you will personalize them by adding your own comments and notes, and checking each section when you feel confident with the material covered.

You will probably also find these summaries useful when you are at the stage of working through the past exams. If you see a particular point being examined that is not summarized here add it to these flashcards. Let us know if you find some key ideas that are missing.

Good luck with your studying.

SURVIVAL MODELS

X is the *random lifetime of a newborn*. There are a number of equivalent ways of specifying the distribution:

1. The *probability density function*: $f_X(x)$. It is not a probability, but $f_X(x)dx \approx \Pr(x \le X \le x + dx)$.

2. The *distribution function*: $F_X(x) = \Pr(X \le x)$ is the probability that the death of a newborn life occurs by age *x*.

3. The *survival function*: $s_X(x) = \Pr(X > x)$ is the probability that a newborn life is surviving at age *x*.

4. The *life table*: l_x is the expected number of survivors at age *x* from a group of l_0 newborn lives.

5. The *force of mortality*: $\mu(x)$ is an instantaneous measure of mortality at age *x*.

Key properties and relations for the PDF and CDF

1. $F_X(x)$ is continuous and non-decreasing with $F_X(0)=0$ and $F_X(\omega)=1$.

2. $f_X(x)$ is non-negative and continuous (or possibly piecewise continuous) on $[0, \omega]$.

3. Relations:

$$F_X(x) = \int_0^x f_X(y)\,dy$$

$$f_X(x) = F'_X(x) \quad \text{(where it exists)}$$

$$\Pr(a \le X \le b) = F_X(b) - F_X(a) = \int_a^b f_X(x)\,dx$$

Key properties and relations for the survival function

1. $s_X(x)$ is continuous and non-increasing with $s_X(0)=1$ and $s_X(\omega)=0$.

2. Relations:

$$s_X(x) = 1 - F_X(x) \text{ and } f_X(x) = -s'_X(x)$$

$$s_X(x) = \int_x^{\omega} f_X(y)\,dy$$

$$\Pr(a \le X \le b) = \int_a^b f_X(x)\,dx = s_X(a) - s_X(b)$$

Key properties and relations for the life table function

1. l_x is the expected number of survivors at age x from a group of l_0 newborn lives.

2. $l_x = l_0\, s_X(x)$ is continuous and non-increasing with $l_\omega = 0$.

3. $s_X(x) = \dfrac{l_x}{l_0}$

Key properties and relations for the force of mortality

1. $\mu(x)$ is non-negative and piece-wise continuous.

2. $\mu(x) = \dfrac{f_X(x)}{s_X(x)} = -\dfrac{s'_X(x)}{s_X(x)} = -\left(\ln\left(s_X(x)\right)\right)' = -\dfrac{l'_x}{l_x}$

3. $s_X(x) = \exp\left(-\int_0^x \mu(y)\,dy\right)$ and $\int_0^{\omega} \mu(y)\,dy = \infty$ in order that $s_X(\omega) = 0$

4. $\mu(x) \cdot \Delta x \approx \Pr\left(X \le x + \Delta x \mid X \ge x\right)$

5. Shortcut: $\mu(x) = c\dfrac{p'(x)}{p(x)} \Rightarrow s_X(x) = \left(\dfrac{p(0)}{p(x)}\right)^c$

Example of thinning

- An insurer wishes to institute an ordinary deductible of d per loss. Let C_1 denote the category of losses that exceed the deductible, and let C_2 denote the category of losses that are less than or equal to the deductible: $p_1 = \Pr(X > d)$. Then $N_1(t)$ is a Poisson process with rate $\lambda_1 = \lambda p_1 = \lambda \Pr(X > d)$. This is the frequency of *payment* events for the insurer over a period of t years.
- Losses could be categorized by *types*. Suppose we have losses against car insurance policies and we want to eliminate broken windshield losses from coverage when 3% of claims are for broken windshields. To know the frequency of losses after this elimination define category C_1 for windshield losses, and category C_2 for all other types of losses. The Poisson process $N_2(t)$ would then have a rate equal to $\lambda_2 = \lambda p_2 = 0.97\lambda$.

Non-homogeneous Poisson process

Denote the *process rate* at time t for a non-homogeneous Poisson process by $\lambda(t)$. The *mean value function* for this process is defined by $m(t) = \int_0^t \lambda(s)\,ds$.