SOA Exam M
Financial Economics

Flashcards

Spring 2009 exams

Key concepts

Important formulas

Efficient methods

Advice on exam technique

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HOW TO USE THESE FLASHCARDS

These flashcards are designed to help you to prepare efficiently in the run-up to the Financial Economics segment (**MFE**) of the Course M exam of the Society of Actuaries and Exam 3F of the Casualty Actuarial Society. They include conceptual ideas, key formulas and techniques for efficient problem solving. This subject includes a number of topics that require first principles reasoning as well as a fair amount of computation. So don't look at the lists of formulas as simply being memorization work. There are often simple intuitive ideas that underlie the formulas as well as basic mathematical reasons why they are correct. Strive to understand and learn the key relations from these points of view and your knowledge will not be the superficial type that may collapse under the stress of taking the examination. The more that you understand, the easier it becomes to retain the key ideas and write them down quickly and accurately.

We have designed the flashcards so that they can be carried conveniently and read frequently in the final run-up to the exam, *eg* when commuting to work. We hope that you will personalize them by adding your own comments and notes, and checking each section when you feel confident with the material covered.

You will probably also find these summaries useful when you are at the stage of working through the past exams. If you see a particular point being examined that is not summarized here add it to these flashcards. Let us know if you find some key ideas that are missing.

Good luck with your studying.

INTRODUCTION TO OPTIONS

Basic notation

1. T = time of expiration of the option

2. t = current time

3. S_t = underlying asset price at time t

4. K = strike price of the option

5. r = continuously-compounded risk-free interest rate

6. σ = volatility of the underlying asset

7. δ = continuously-paid yield on a stock

Call options

A standard *call option* provides its owner with the right (but with no obligation) to purchase the underlying asset for a fixed strike price (K).

Call option payoff $= \max(S_T - K, 0)$

$S_t > K \Leftrightarrow$ in-the-money call
$S_t = K \Leftrightarrow$ at-the-money call
$S_t < K \Leftrightarrow$ out-of-the-money call

Put options

A standard *put option* provides its owner with the right to sell the underlying asset for a fixed strike price (K).

Put option payoff $= \max(K - S_T, 0)$

$S_t < K \Leftrightarrow$ in-the-money put
$S_t = K \Leftrightarrow$ at-the-money put
$S_t > K \Leftrightarrow$ out-of-the-money put

European versus American options

European options can only be exercised at the time that the option expires.

American options can be exercised at any time on or before the expiration of the option.

INTRODUCTION TO OPTIONS

Forward prices

A *forward price* is a price agreed upon now for a transaction scheduled to take place at a specified time in the future.

If the forward price agreed upon now for the purchase of a share of stock at time T is $F_{0,T}$, then:

Payoff to the buyer of the forward $= S_T - F_{0,T}$

Payoff to the seller of the forward $= F_{0,T} - S_T$

The payoff to a forward agreement can be positive or negative, but vanilla call and put options never have negative payoffs.

The forward price for a share depends on the dividends paid to the stockholders:

1. No dividends: $F_{0,T} = S_0 e^{rT}$

2. Discrete dividends: $F_{0,T} = \left[S_0 - PV_{0,T}(Div) \right] e^{rT}$

$$= S_0 e^{rT} - FV_{0,T}(Div)$$

where $PV_{0,T}(Div) = \sum_{i=1}^{n} e^{-rt_i} D_{t_i}$ and $FV_{0,T}(Div) = \sum_{i=1}^{n} e^{r(T-t_i)} D_{t_i}$

3. Continuous dividends: $F_{0,T} = S_0 e^{(r-\delta)T}$

Prepaid forwards

A *prepaid forward price* is the amount that can be paid now in order to receive an asset at a specified time in the future.

$$F_{0,T}^{P} = F_{0,T} e^{-rT} = PV_{0,T}(F_{0,T})$$

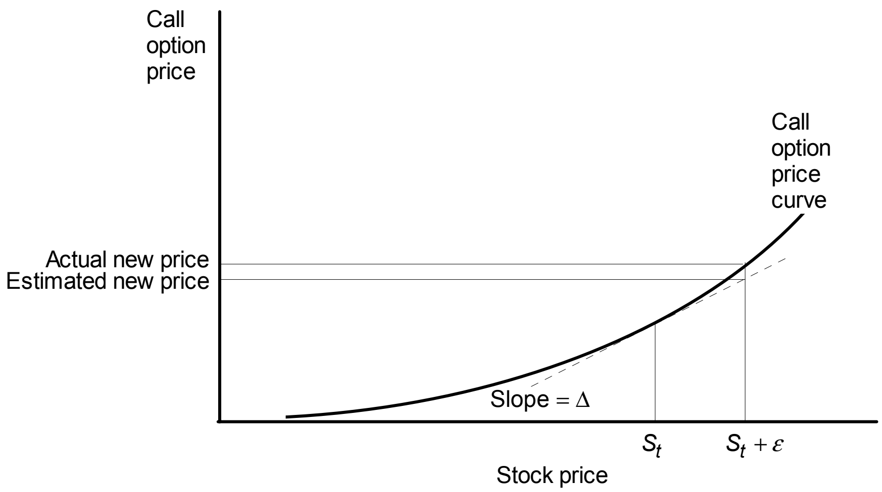
The prepaid forward price, $F_{0,T}^{P}$, paid at time 0 for an asset to be delivered at time T depends on the dividends paid to the stockholders:

1. No Dividends: $F_{0,T}^{P} = S_0$

2. Discrete dividends: $F_{0,T}^{P} = S_0 - PV_{0,T}(Div)$

3. Continuous dividends: $F_{0,T}^{P} = S_0 e^{-\delta T}$

Delta as the slope of the price curve



If the stock price increases by ε, we can use delta to estimate the new price. As shown in the graph above, using delta alone underestimates the new price. This is because the graph of the call option price is convex. Using gamma can improve our estimate of the new price.

Delta-hedging algorithm

1. Determine the delta of the position to be hedged:

$$\Delta_{Position} = \sum_{i=1}^{n} \omega_i \Delta_i$$

where ω_i = quantity of option i in the position to be hedged

2. Purchase $-\Delta_{Position}$ shares of the underlying stock.

This is equivalent to selling $\Delta_{Position}$ shares.

3. Borrow or lend at the risk-free rate of return.

The amount to borrow is the cost of the stock purchase minus the proceeds from the position to be hedged:

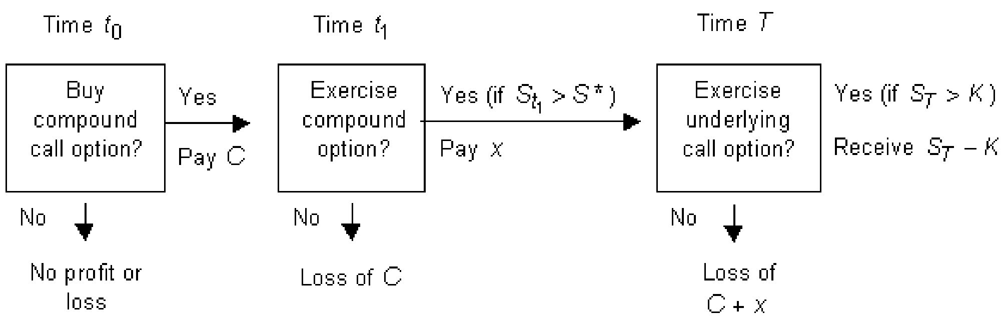
$$-S_t \Delta_{Position} + \sum_{i=1}^{n} \omega_i (\text{Option Price})_i$$

This is equivalent to lending:

$$S_t \Delta_{Position} - \sum_{i=1}^{n} \omega_i (\text{Option Price})_i$$

Payoff function for a compound option

For a call option on a call option, where both options are European, the cashflows are:



where S^* is the asset price such that $C(S^*, K, T-t_1) = x$.

For a compound put option, S^* would be defined as the asset price for which $P(S^*, K, T-t_1) = x$, and we would choose to exercise if $S_{t_1} < S^*$.

For this compound option, the overall payoff function is:

$$-C + I(S_{t_1} > S^*)[-x + \max(S_T - K, 0)]$$

where I is an indicator function.

Put-call parity for compound options

The prices of a European compound call option and a European compound put option on the same underlying option, both with the same strike price x and expiring at the same time t_1, satisfy:

$$\begin{bmatrix} \textit{Value of} \\ \textit{Compound Call Option} \end{bmatrix} - \begin{bmatrix} \textit{Value of} \\ \textit{Compound Put Option} \end{bmatrix} = \begin{bmatrix} \textit{Value of} \\ \textit{Underlying Option} \end{bmatrix} - xe^{-rt_1}$$

