# *SOA Exam M Financial Economics*

# *Flashcards*

## *Fall 2011 exams*

***Key concepts***

***Important formulas***

***Efficient methods***

***Advice on exam technique***

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## *HOW TO USE THESE FLASHCARDS*

These flashcards are designed to help you to prepare efficiently in the run-up to the Financial Economics segment (**MFE**) of the Course M exam of the Society of Actuaries and Exam 3F of the Casualty Actuarial Society. They include conceptual ideas, key formulas and techniques for efficient problem solving. This subject includes a number of topics that require first principles reasoning as well as a fair amount of computation. So don't look at the lists of formulas as simply being memorization work. There are often simple intuitive ideas that underlie the formulas as well as basic mathematical reasons why they are correct. Strive to understand and learn the key relations from these points of view and your knowledge will not be the superficial type that may collapse under the stress of taking the examination. The more that you understand, the easier it becomes to retain the key ideas and write them down quickly and accurately.

We have designed the flashcards so that they can be carried conveniently and read frequently in the final run-up to the exam, *eg* when commuting to work. We hope that you will personalize them by adding your own comments and notes, and checking each section when you feel confident with the material covered.

You will probably also find these summaries useful when you are at the stage of working through the past exams. If you see a particular point being examined that is not summarized here add it to these flashcards. Let us know if you find some key ideas that are missing.

Good luck with your studying.


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**Black-Scholes formulas using prepaid forward prices**

$$C_{Eur} = F_{0,T}^{P}(S)N(d_1) - F_{0,T}^{P}(K)N(d_2)$$

$$P_{Eur} = F_{0,T}^{P}(K)N(-d_2) - F_{0,T}^{P}(S)N(-d_1)$$

where:

$$d_1 = \frac{\ln\left(\frac{F_{0,T}^{P}(S)}{F_{0,T}^{P}(K)}\right) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

**Options on dividend-paying stocks**

Use $F_{0,T}^{P}(S) = \begin{cases} Se^{-\delta T} & \text{for fixed dividend yield} \\ S - PV_{0,T}(Div) & \text{for discrete dividend payments} \end{cases}$

$$F_{0,T}^{P}(K) = Ke^{-rT}$$

Put price based on put-call parity and prepaid forward prices:

$$P_{Eur} = C_{Eur} + F_{0,T}^{P}(K) - F_{0,T}^{P}(S)$$

**Options on currencies**

$x_0$ = the price of the asset
$r_f$ = the foreign interest rate (treated like the dividend yield)

Use $F_{0,T}^{P}(x) = x_0 e^{-r_f T}$.

**Options on futures**

Use $F_{0,T}^{P}(S) = Fe^{-rT}$ where $F$ is the current futures price.

Alternatively, use the Black formula:

$$C_{Eur} = Fe^{-rt}N(d_1) - Ke^{-rt}N(d_2)$$

where:

$$d_1 = \frac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

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## Option Greeks

**Delta**

$$\Delta = \frac{\partial(\text{Option value})}{\partial(\text{Stock price})}$$

For a *European call option*: $\Delta = e^{-\delta T}N(d_1)$

A European call option can be *replicated* by purchasing $\Delta$ shares and borrowing $Ke^{-rT}N(d_2)$.

For a *European put option*: $\Delta = e^{-\delta T}N(d_1) - e^{-\delta T} = -e^{-\delta T}N(-d_1)$

A European put option can be *replicated* by selling $-\Delta$ shares and lending $Ke^{-rT}N(-d_2)$.

| | Call option delta | Put option delta |
|---|---|---|
| **Sign** | positive | negative |
| **Deep in-the-money** | approaches 1 | approaches –1 |
| **Deep out-of-the-money** | approaches 0 | approaches 0 |

**Gamma**

$$\Gamma = \frac{\partial^2(\text{Option value})}{\partial(\text{Stock price})^2} = \frac{\partial(\Delta)}{\partial(\text{Stock price})}$$

When a position has positive gamma across all of the possible stock prices, the position is said to be *convex*. A position that includes selling calls and/or puts may not be convex.

$\Gamma_{call} = \Gamma_{put}$ for European options with same $K$ and $T$.

| | Call option gamma | Put option gamma |
|---|---|---|
| **Sign** | positive | positive |
| **Deep in-the-money** | approaches 0 | approaches 0 |
| **Deep out-of-the-money** | approaches 0 | approaches 0 |


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## Market-makers

A *market-maker* for an option stands ready to sell the option for an ask price and buy it for a bid price. The bid price is less than the ask price, allowing the market-maker to earn a profit.

## Market-maker risk

If the number of options sold by the market-maker does not match the number purchased by the market-maker, then the market-maker will be exposed to *risk* arising from potential changes in the value of the option.

Positions are *marked-to-market* on a daily basis.

Market-makers can control risk by *delta-hedging*.

### Borrowing and lending

Delta-hedging begins with a position to be hedged.

When hedging the sale of an option, delta-hedging requires the purchase of $\Delta$ shares of stock. In general, the proceeds from selling the option will not be the same as the amount needed for buying $\Delta$ shares of stock.

If an injection of capital is required, the capital can be obtained through borrowing or from the market-maker's own funds. In either case, the capital earns the risk-free rate.

If the cost of establishing the delta-hedged position is negative, then funds are available for lending. These funds also earn interest at the risk-free rate.

**Delta of the position to be hedged**

$$\Delta_{Position} = \sum_{i=1}^{n} \omega_i \Delta_i$$

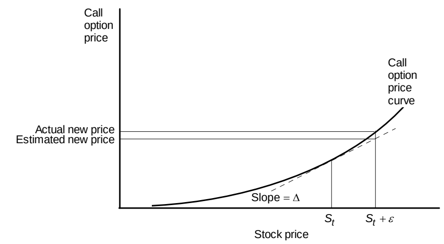
where:

$\omega_i$ = quantity of option $i$ owned





**Delta as the slope of the price curve**




If the stock price increases by $\varepsilon$, we can use delta to estimate the new price. As shown in the graph above, using delta alone underestimates the new price. This is because the graph of the call option price is convex. Using gamma can improve our estimate of the new price.

**Delta-hedging algorithm**

1. Determine the delta of the position to be hedged:

   $$\Delta_{Position} = \sum_{i=1}^{n} \omega_i \Delta_i$$

   where $\omega_i$ = quantity of option $i$ in the position to be hedged

2. Purchase $-\Delta_{Position}$ shares of the underlying stock.

   This is equivalent to selling $\Delta_{Position}$ shares.

3. Borrow or lend at the risk-free rate of return.

   The amount to borrow is the cost of the stock purchase minus the proceeds from the position to be hedged:

   $$-S_t \Delta_{Position} + \sum_{i=1}^{n} \omega_i (\text{Option Price})_i$$

   This is equivalent to lending:

   $$S_t \Delta_{Position} - \sum_{i=1}^{n} \omega_i (\text{Option Price})_i$$