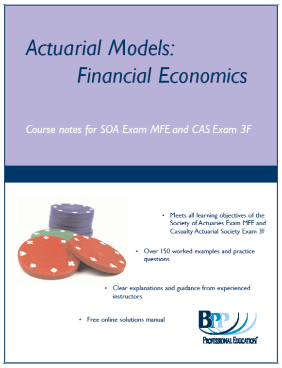

Financial economics (MFE)

Published by BPP Professional Education

Solutions to practice questions – Chapter 5

Solution 5.1

The formula for delta for a put option is:

$$\Delta = -e^{-\delta T} N(-d_1)$$

Here we have:

$$S = K = 10,\ \sigma = 0.35,\ r = 0.05,\ T = 1,\ \delta = 0.02$$

The value of d_1 is:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \delta + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{10}{10}\right) + \left(0.05 - 0.02 + \frac{1}{2} \times 0.35^2\right)(1)}{0.35\sqrt{1}} = 0.2607$$

From the Tables:

$$N(-d_1) \approx N(-0.26) = 0.3974$$

So the delta of the put option is:

$$\Delta = -e^{-\delta T} N(-d_1) = -e^{-0.02(1)} \times 0.3974 = -0.3895$$

If the trader has a long holding of 2,000 put options, the portfolio delta will be $2,000 \times (-0.3895) = -779$. To create a delta-hedged position, the trader needs to buy 779 shares.

We can check that this position is now delta-hedged by calculating the overall delta, which is:

$$2,000\Delta_{put} - 779\Delta_{stock} = 2,000 \times (-0.3895) + 779 \times 1 = 0$$

Solution 5.2

In a portfolio consisting of a long position in call options, the delta approximation will UNDER-estimate the revised value of the portfolio because no CONVEXITY adjustment has been applied.

A more accurate approximation will be obtained if the second-order Greek GAMMA is included in the calculation.

The approximation can be improved still further by incorporating THETA, which takes into account time decay.

The equation for the new price, taking into account all three adjustments, is: $C(S_{t+h}) \approx C(S_t) + \varepsilon\Delta_t + \frac{1}{2}\varepsilon^2\Gamma_t + h\theta_t$.

Solution 5.3

Using the delta-gamma-theta approximation, the new option price is estimated to be:

$$\begin{aligned} C(S_{t+h}) &\approx C(S_t) + \varepsilon\Delta_t + \tfrac{1}{2}\varepsilon^2\Gamma_t + h\theta_t \\ &= 1.20 + (-0.50)(-0.389) + \tfrac{1}{2}(-0.50)^2(0.108) + \tfrac{1}{12}(-0.484) \\ &= 1.37 \end{aligned}$$

To calculate the new price accurately, we need to use the formula:

$$P_{Eur} = Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$$

The new parameter values are:

$$S = 9.50,\ K = 10,\ \sigma = 0.35,\ r = 0.05,\ T = \tfrac{11}{12},\ \delta = 0.02$$

The values of d_1 and d_2 are:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \delta + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{9.50}{10}\right) + \left(0.05 - 0.02 + \frac{1}{2}\times 0.35^2\right)\left(\frac{11}{12}\right)}{0.35\sqrt{\frac{11}{12}}} = 0.0965$$

and $\quad d_2 = d_1 - \sigma\sqrt{T} = 0.0965 - 0.35\sqrt{\tfrac{11}{12}} = -0.2386$

From the Tables:

$$N(d_1) \approx N(0.10) = 0.5398$$

$$\Rightarrow \quad N(-d_1) = 1 - N(d_1) = 1 - 0.5398 = 0.4602$$

$$N(d_2) \approx N(-0.24) = 0.4052$$

$$\Rightarrow \quad N(-d_2) = 1 - N(d_2) = 1 - 0.4052 = 0.5948$$

So the price of the put option is:

$$\begin{aligned} P_{Eur} &= Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1) \\ &= 10e^{-0.05\times\frac{11}{12}} \times 0.5948 - 9.50e^{-0.02\times\frac{11}{12}} \times 0.4602 \\ &= 1.39 \end{aligned}$$

This answer is close to the approximate answer calculated using the delta-gamma-theta approximation.

If we carry out these calculations more accurately, without approximating the N functions, the new option price based on the new parameter values is 1.3715. The estimated price calculated using the delta-gamma-theta approximation is 1.3677. These two answers are very close, but they are not exactly the same because the values of ε and h are not infinitesimally small.

Solution 5.4

The Black-Scholes (partial differential) equation is:

$$rC(S_t) = \frac{1}{2}\sigma^2 S_t^2 \Gamma_t + rS_t \Delta_t + \theta_t$$

Substituting the numerical values given for the parameters:

$$0.05(60,000) = 0.5(0.2)^2(1050)^2(1.25) + 0.05(1050)(240) + \theta_t$$

$$3000 = 27,562.5 + 12,600 + \theta_t$$

So: $\theta_t = -37,162.5$

This corresponds to a daily rate of time decay of:

$$37,162.5/250 = 148.65 \quad \text{per trading day}$$

Solution 5.5

The general formula is:

$$\sigma(R_h) = \frac{1}{\sqrt{n}} \times \frac{1}{\sqrt{2}}\left(S^2\sigma^2\Gamma h\right)$$

Here $h = \frac{1}{4}$ and $n = 13$.

So: $$\sigma(R_h) = \frac{1}{\sqrt{n}} \times \frac{1}{\sqrt{2}}\left(S^2\sigma^2\Gamma h\right) = \frac{S^2\sigma^2\Gamma}{4\sqrt{13}\sqrt{2}} = 0.049S^2\sigma^2\Gamma$$

Solution 5.6

The original position has the following characteristics:

Original position:		Delta = 250	Gamma = 50

We first need to make the position gamma-neutral. To do this we need to sell $\frac{50}{0.05} = 1,000$ of Option A. The effect of this adjustment to the portfolio is as follows:

Original position:		Delta = 250	Gamma = 50
Sell 1,000 options:	Cost = –2,000	Delta = –500	Gamma = –50
Gamma-neutral position:	Cost = –2,000	Delta = –250	Gamma = 0

We then need to make the position delta-neutral. To do this we need to buy 250 units of the underlying asset. The effect of this adjustment to the portfolio is as follows:

Gamma-neutral position:	Cost = –2,000	Delta = –250	Gamma = 0
Buy 250 underlying:	Cost = 12,500	Delta = +250	Gamma = 0
Delta-neutral position:	Cost = 10,500	Delta = 0	Gamma = 0

The cost of these adjustments is 10,500, which we will need to borrow.
