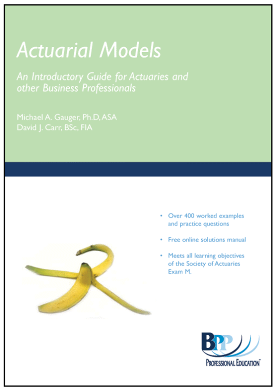

Actuarial models

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Published by BPP Professional Education

Solutions to practice questions – Chapter 8

Solution 8.1

The extended equivalence principle equates the actuarial present value at issue of the expense load premiums with the actuarial present value at issue of the benefits and the expenses.

Suppose that the level annual expense loaded premium is G and the level annual benefit premium is P. If these premiums are paid annually for life, then we have:

$$
\begin{aligned}
&P\,\ddot{a}_x = A && (A \text{ is the APV of benefits})\\
&G\ddot{a}_x = A + E && (E \text{ is the APV of expenses})\\
&\Rightarrow (G-P)\ddot{a}_x = E
\end{aligned}
$$

The premium difference $L = G - P$ is a level annual amount whose APV at issue is equal to the APV at issue of the expenses.

Solution 8.2

$$
\begin{aligned}
&\text{First year expense:} && 0.05G + 5 + 2\times 10 = 0.05G + 25\\
&\text{Renewal year expense:} && 0.02G + 2 + 1\times 10 = 0.02G + 12
\end{aligned}
$$

Solution 8.3

The recurring annual expenses occur each year that the policy remains in force. So they can be viewed as a level life annuity due of $0.02G + 12$, plus an extra amount $0.03G + 13$ at time zero. So the actuarial present value at issue of the recurring annual expenses is:

$$(0.02G + 12)\ddot{a}_{40} + 0.03G + 13$$

Solution 8.4

$$G\,\ddot{a}_{40} = 10{,}000\,A_{40} + (0.02G+12)\ddot{a}_{40} + 0.03G+13$$
$$L\,\ddot{a}_{40} = (0.02G+12)\ddot{a}_{40} + 0.03G+13$$

These equations should make it clear that G must be computed first. The reason is that percent of premium expenses vary with the premium. The actual expense amounts are only known after you compute G.

Solution 8.5

Since the number of thousands of insurance is the face value divided by 1,000, the amount of the settlement expense is:

$$50 + 10 \times \frac{10{,}000}{1{,}000} = 150$$

It is a one-time cost at the end of the year of death of (40), so the actuarial present value at issue is:

$$150\,A_{40}$$

Solution 8.6

$$G\,\ddot{a}_{40} = 10{,}150\,A_{40} + (0.02G+12)\ddot{a}_{40} + 0.03G+13$$
$$L\,\ddot{a}_{40} = (0.02G+12)\ddot{a}_{40} + 0.03G+13 + 150\,A_{40}$$

Solution 8.7

Recall that with de Moivre's law there is a shortcut method for calculating A_x as $a_{\overline{\omega-x}|}/(\omega-x)$:

$$A_{40} = \frac{a_{\overline{50}|}}{50} = \frac{18.25593}{50} = 0.36512 \;\Rightarrow\; \ddot{a}_{40} = 13.33251$$
$$10{,}000\,P_{40} = \frac{10{,}000\,A_{40}}{\ddot{a}_{40}} = 273.85577$$

$$G\,\ddot{a}_{40} = 10{,}150\,A_{40} + (0.02G+12)\ddot{a}_{40} + 0.03G+13 \;\Rightarrow$$
$$G = \frac{10{,}150\,A_{40} + 12\,\ddot{a}_{40} + 13}{0.98\,\ddot{a}_{40} - 0.03} = \frac{3{,}878.94300}{13.03586} = 297.55940 \Rightarrow$$
$$L = G - 10{,}000\,P_{40} = 297.55940 - 273.85577 = 23.70363$$

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Solution 8.8

First year expense: $0.05G + 25 = 39.87797$
Renewal year expense: $0.02G + 12 = 17.95119$
Settlement expense: 150

The actuarial present values at issue of the expenses and loadings are:

Expenses: $150\,A_{40} + 17.95119\,a_{40} + 39.87797 = 316.02898$
Loadings: $23.70363\,\ddot{a}_{40} = 316.02898$

Solution 8.9

The benefit reserve at duration 10 is computed as follows:

$$\ddot{a}_{40} = 13.33251 \qquad \text{(See Solution 8.7)}$$

$$\ddot{a}_{50} = \left(1 - A_{50}\right)/\,d$$

$$= \frac{1 - \left(a_{\overline{40}|}/40\right)}{0.05/1.05} = 11.99148 \qquad \text{(de Moivre's law)}$$

$$10{,}000\;{}_{10}V_{40} = 10{,}000\left(1 - \frac{\ddot{a}_{50}}{\ddot{a}_{40}}\right) = 1{,}005.83574$$

Solution 8.10

At duration 10, look into the future and you will see level recurring annual expense equal to 17.95119 and a settlement expense of 150 at the end of the year of death. The level annual loading is 23.70363. (See Solutions 8.7 and 8.8.)

So the expense reserve at duration 10 is computed as follows:

$$\begin{aligned}
{}_{10}V^{\text{expense}} &= APV_{50}\left(\text{future expenses - future loadings}\right)\\
&= 150\,A_{50} + 17.95119\,\ddot{a}_{50} - 23.70363\,\ddot{a}_{50}\\
&= 150\left(1 - \frac{0.05}{1.05}\ddot{a}_{50}\right) - 5.75244\,\ddot{a}_{50}\\
&= 150\left(1 - \frac{0.05}{1.05}\times 11.99148\right) - 5.75244 \times 11.99148\\
&= -4.63370
\end{aligned}$$

So the total reserve at duration 10 is:

$${}_{10}V^{\text{total}} = {}_{10}V^{\text{expense}} + 10{,}000\;{}_{10}V_{40} = -4.63369 + 1{,}005.83574 = 1{,}001.20$$

Solution 8.11

The level annual expense loaded premium is $G = 297.55940$ (see Solution 8.7). The expenses are:

First year expense: 39.87797
Renewal year expense: 17.95119
Settlement expense: 150

We will need the following values based on de Moivre's law with $\omega = 90$:

$$A_{40} = \frac{a_{\overline{50}|}}{50} = 0.36512 \qquad \ddot{a}_{40} = 13.33251$$

$$A_{50} = \frac{a_{\overline{40}|}}{40} = 0.42898 \qquad \ddot{a}_{40} = 11.99148$$

$$v^{10}\ {}_{10}p_{40} = \frac{40}{50 \times 1.05^{10}} = 0.49113$$

$$A^{1}_{40:\overline{10}|} = 0.15443 \qquad \ddot{a}_{40:\overline{10}|} = 7.44313$$

The asset share at duration 10 is a survivor's share of the accumulation at time 10 of past premium less past benefits and expenses:

$$\begin{aligned}
{}_{10}AS(G) &= \frac{1}{v^{10}\ {}_{10}p_{40}} \left(\underbrace{297.55940\,\ddot{a}_{40:\overline{10}|}}_{\text{past premium}} - \underbrace{10{,}150\,A^{1}_{40:\overline{10}|}}_{\substack{\text{past benefits and}\\ \text{settlement expenses}}} - \underbrace{\left(17.95119\,\ddot{a}_{40:\overline{10}|} + (39.87797 - 17.95119)\right)}_{\text{past recurring annual expenses}} \right) \\
&= \frac{1}{0.49113}\left(297.55940 \times 7.44313 - 10{,}150 \times 0.15443 - 17.95119 \times 7.44313 - 21.92678\right) \\
&= 1{,}001.20
\end{aligned}$$

Solution 8.12

If G is now the level annual contract to be determined as a result of some profit target at duration 10, then we have the following as a result of Solutions 8.8 and 8.11:

$$\begin{aligned}
{}_{10}AS(G) &= \frac{1}{v^{10}\ {}_{10}p_{40}} \left(\underbrace{G\,\ddot{a}_{40:\overline{10}|}}_{\text{past premium}} - \underbrace{10{,}150\,A^{1}_{40:\overline{10}|}}_{\substack{\text{past benefits and}\\ \text{settlement expenses}}} - \underbrace{\left((0.02G + 12)\,\ddot{a}_{40:\overline{10}|} + 0.03G + 13\right)}_{\text{past recurring annual expenses}} \right) \\
&= \frac{1}{0.49113}\left(7.26427G - 1{,}669.82973\right) = 14.79091G - 3{,}399.97085
\end{aligned}$$

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Solution 8.13

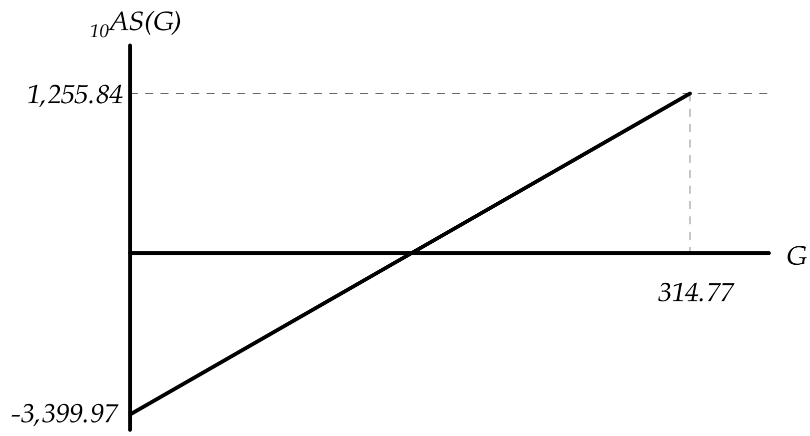
The benefit reserve at duration 10 was seen to be $10{,}000\,{}_{10}V_{40} = 1{,}005.83574$ in Solution 8.9. If the profit target at duration 10 is 250 for an in force policy, then the asset share target at duration 10 is:

$$ {}_{10}AST(G) = 250 + 1{,}005.83574 = 1{,}255.83574 $$

Set this number equal to the actual asset share formula derived in Solution 8.12 and solve the resulting equation:

$$ 1{,}255.83574 = {}_{10}AS(G) = 14.79091G - 3{,}399.97085 \quad \Rightarrow \quad G = 314.77 $$

Solution 8.14



Solution 8.15

The profit target at duration 10 is:

$$ 0.05G\,\ddot{s}_{40:\overline{10|}} $$

Since liability is measured by the benefit reserve at duration 10, the asset share target at duration 10 is:

$$ {}_{10}AST(G) = 0.05G\,\ddot{s}_{40:\overline{10|}} + 1{,}005.83574 = 0.05G\,\frac{\ddot{a}_{40:\overline{10|}}}{v^{10}\,{}_{10}p_{40}} + 1{,}005.83574 $$

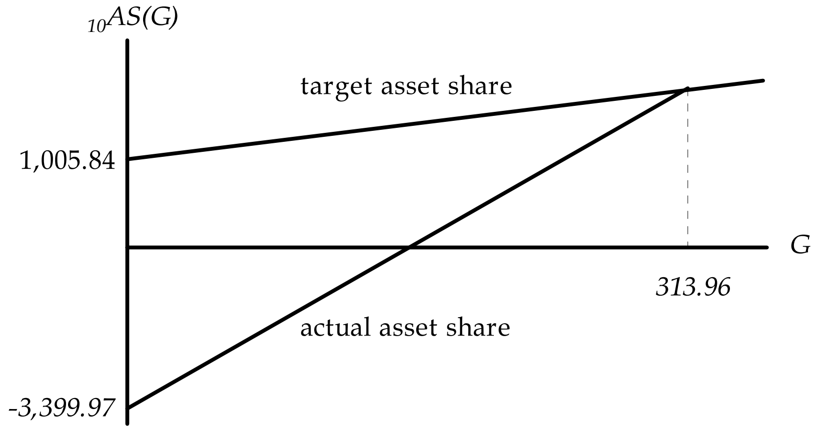
$$ = 0.75775G + 1{,}005.83574 \qquad \text{(some numbers here are from Solutions 8.9 and 8.11)} $$

Set this linear expression equal to the actual asset share formula derived in Solution 8.12 and solve the resulting equation:

$$ 0.75775G + 1{,}005.83574 = {}_{10}AST(G) = {}_{10}AS(G) = 14.79091G - 3{,}399.97085 $$

$$ \Rightarrow \quad G = \frac{4{,}405.80659}{14.03315} = 313.96 $$

Solution 8.16



Solution 8.17

The recursive asset share relation in the aggregate deterministic form is:

$$\underbrace{\Big(\underbrace{l_{x+k}\ {}_kAS}_{{}_kEF} + \underbrace{l_{x+k}\big(G(1-f_k)-e_k\big)}_{\substack{\text{premium less expenses}\\ \text{for year } k+1}}\Big)(1+i)}_{\text{beginning fund plus premium less expenses plus interest}} - \underbrace{d_{x+k}(F+S)}_{\substack{\text{benefits and}\\ \text{settlement expense}}} = \underbrace{{}_{k+1}EF = l_{x+k+1}\ {}_{k+1}AS}_{\text{ending fund for year } k+1}$$

Now divide both sides of this equation by l_{x+k} to obtain a relation for a single policy in force at duration k:

$$\big({}_kAS + G(1-f_k)-e_k\big)(1+i) - q_{x+k}(F+S) = p_{x+k}\ {}_{k+1}AS$$

Now adapt this equation to year 11 for the insurance in Question 8.7. All of the numbers we need are available in Solutions 8.7, 8.8, and 8.11:

$${}_{10}AS = 1{,}001.20 \qquad F+S = 10{,}150 \qquad G = 297.55940$$
$$e_{10} = 12 \qquad f_{10} = 0.02$$

$$\begin{aligned} p_{50}\ {}_{11}AS &= \big({}_{10}AS + G(1-f_{10})-e_{10}\big)(1+0.05) - q_{50}(F+S) \\ &= (1{,}001.20 + 297.55940\times 0.98 - 12)(1.05) - q_{50}\,10{,}150 \end{aligned}$$

Solution 8.18

With $l_x = 90 - x$, we have $p_{50} = l_{51}/l_{50} = 39/40$ and $q_{50} = 1/40$. As a result, we have:

$$\begin{aligned} \frac{39}{40}\times {}_{11}AS &= \big({}_{10}AS + G(1-f_{10})-e_{10}\big)(1+0.05) - q_{50}(F+S) \\ &= (1{,}001.20 + 297.55940\times 0.98 - 12)(1.05) - 10{,}150\times\frac{1}{40} \\ &\Rightarrow\ {}_{11}AS = 1{,}119.08 \end{aligned}$$

Solution 8.19

The total reserve at duration 11 is the same as the 11-th asset share computed in Solution 8.18. This is simply the fact that when you use expense loaded premiums in a spreadsheet for the aggregate deterministic model, the ending fund in year 11 is the same as aggregate total reserves. Divide this by the number of in force policies, and you have the 11-th asset share equal to the 11-th total reserve for an in force policy.

Solution 8.20

Since we have the total reserve for year 11, we could either calculate the expense reserve directly, or we could calculate it indirectly as the difference between the 11-th year total reserve and the 11-th year benefit reserve:

$$10,000\ {}_{11}V_{40} = 10,000\left(1 - \frac{\ddot{a}_{51}}{\ddot{a}_{40}}\right) = 1,121.72$$

since:

$$\ddot{a}_{40} = 13.33251 \qquad \text{(Solution 8.11)}$$

$$\ddot{a}_{51} = \frac{1 - A_{51}}{d} = \frac{1 - a_{\overline{39}|}/39}{d} = 11.83698$$

As a result, we have:

$$\begin{aligned} {}_{11}V^{\text{expense}} &= {}_{11}V^{\text{total}} - 10,000\ {}_{11}V_{40} \\ &= {}_{11}AS - 10,000\ {}_{11}V_{40} \\ &= 1,119.08 - 1,121.72 = -2.64 \end{aligned}$$