Solutions to practice questions – Study Session 13

Solution 13.1

If $F_{0,T}^{P} \geq S_0$:

	Cash flows	
	$t=0$	$t=T$
Buy asset	$-S_0$	S_T
Short forward	$F_{0,T}^{P}$	$-S_T$
Total	$F_{0,T}^{P} - S_0$	0

If $F_{0,T}^{P} < S_0$:

	Cash flows	
	$t=0$	$t=T$
Short asset	S_0	$-S_T$
Buy forward	$-F_{0,T}^{P}$	S_T
Total	$S_0 - F_{0,T}^{P}$	0

Since we have achieved arbitrage (positive cash flow with no risk) the assumptions must be false and $S_0 = F_{0,T}^{P}$

Solution 13.2

$F_{0,1}^{P} = \$75 - \$1e^{-0.08(4/12)} - \$1e^{-0.08(10/12)} = \$73.09$

If dividends were less certain then the inherent risk could be reflected by using a discount rate greater than 8%.

Solution 13.3

Sell the observed prepaid forward for \$47.75 and buy the fairly priced prepaid forward for \$47.60, for a net initial cash inflow of $\$47.75 - \$47.60 = \$0.15$ at no risk.

This may seem like small potatoes but, if the transaction can be repeated, the profits can begin to become significant.

Solution 13.4

The payoff on a call option is:

$$Max(0, S_T - K)$$

As $K \to 0$ the option becomes every more likely to be exercised and the expected payoff tends towards S_T. The fair value of such an option at the outset is then:

$$F_{0,T}^P - PV(K)$$

Which is the value of a prepaid forward.

➔ *Such options exist and are termed low exercise price options (LEPOs).*

Solution 13.5

If *r* is the risk-free rate, then the forward price of the bond is given by:

$$F_{0,2} = 101.08 = 95\,(1+r)^2 - 3(1+r)^{½} - 3(1+r)^{1½}$$

We can use the cash flow worksheet on the BAII Plus calculator. Working in half-years, we set:

[CF0]= –95 , [C01]=3, [F01]=1,[C02]=0, [F01]=1,[C03]=3, [F01]=1,[C04]=101.08, [F04]=1

The [IRR][CPT] gives us 3.13% per half-year (*ie* 6.358% p.a.)

$$\begin{aligned} F_{0,2} &= 95\,(1.0313)^4 - 3(1.0313)^1 - 3(1.0313)^3 \\ &= 101.08 \end{aligned}$$

We conclude that the annual effective risk-free rate is 6.358%.

The corresponding prepaid forward price is equal to:

$$F_{0,2}^P = \frac{F_{0,2}}{1.06358^2} = 89.36$$

Solution 13.6

The size of each S&P 500 index futures contract is $250, so the multiplier for 6 futures contracts is $6 \times 250 = 1,500$.

Assuming the margin account is marked-to-market at the end of each month, the margin balance will develop as follows, remembering that the investor has a short position:

Month	Multiplier	Futures price	Price change	Margin balance
0	1,500	1,295	-	388,500.00
1	1,500	1,305	+10	375,447.36
2	1,500	1,287	–18	404,329.30
3	1,500	1,337	+50	331,356.01

The margin balance at time 0 is:

$$B_0 = 1,500 \times 1,295 \times 0.2 = 388,500.00$$

The margin balance after 1 month is:

$$B_1 = 388,500 \times e^{0.06(1/12)} - 10 \times 1,500 = 375,447.36$$

The margin balance after 2 months is:

$$B_2 = 375,447.36 \times e^{0.06(1/12)} + 18 \times 1,500 = 404,329.30$$

The margin balance after 3 months is:

$$B_3 = 404,329.30 \times e^{0.06(1/12)} - 50 \times 1,500 = 331,356.01$$

So, the profit earned after 3 months is:

$$331,356.01 - 388,500.00 \times e^{0.06(3/12)} = -63,015.41$$

ie a *loss* of about $63,015.

Solution 13.7

The correct answer is B.

Statement I is false. The price of a forward contract on a stock that pays no dividends is the initial stock price accumulated for T years at the risk-free interest rate r. The price of a prepaid forward contract is the present value at time 0 of the price of a forward contract (discounted at the risk-free interest rate), which in this case is just the initial stock price.

Statement II is true.

Statement III is false. To create a synthetic short forward contract, an investor would need to short sell $e^{-\delta T}$ shares of stock and lend $S_0 e^{-\delta T}$ at the risk-free rate.