Solutions to practice questions – Study Session 12

Solution 12.1

	Two long 200 at $10		One short 217.35 at $20		Corn asset	Hedged position	
Corn price	Payoff	Profit	Payoff	Profit	Payoff/Profit	Payoff	Profit
160	80	59.60392	–57.35	–36.95392	–15	7.65	7.65
165	70	49.60392	–52.35	–31.95392	–10	7.65	7.65
170	60	39.60392	–47.35	–26.95392	–5	7.65	7.65
175	50	29.60392	–42.35	–21.95392	0	7.65	7.65
180	40	19.60392	–37.35	–16.95392	5	7.65	7.65
185	30	9.60392	–32.35	–11.95392	10	7.65	7.65
190	20	–0.39608	–27.35	–6.95392	15	7.65	7.65
195	10	–10.39608	–22.35	–1.95392	20	7.65	7.65
200	0	–20.39608	–17.35	3.04608	25	7.65	7.65
205	0	–20.39608	–12.35	8.04608	30	17.65	17.65
210	0	–20.39608	–7.35	13.04608	35	27.65	27.65
215	0	–20.39608	–2.35	18.04608	40	37.65	37.65
220	0	–20.39608	0	20.39608	45	45	45
225	0	–20.39608	0	20.39608	50	50	50
230	0	–20.39608	0	20.39608	55	55	55

Solution 12.2

Note: In earlier versions of the notes the cost of production was omitted from the question. It should have been stated as being 100 per unit.

If the price in 6 months' time is 90, then the (after-tax) profit is:

$$90 - 100 = -10$$

(This is represented by point A on the graph in section 12.4 of the notes.)

If the price in 6 months' time is 115, then the (after-tax) profit is:

$$\left(\frac{115 - 100}{2}\right) = 7.5$$

(Point C on the graph in section 12.4 of the notes.)

So, the expected after-tax profit without hedging (point D on the graph) is:

$\left(\frac{-10 + 7.5}{2}\right) = -1.25$, *ie* without hedging there is an expected loss

With hedging the price in 6 months' time can be guaranteed to be (point P on the graph):

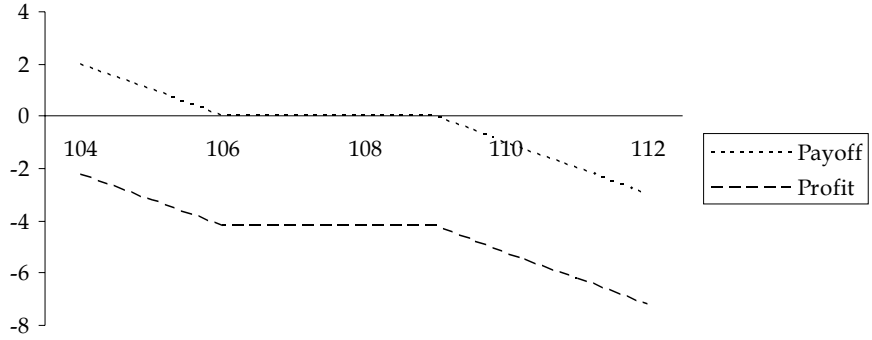
$$\left(\frac{90 + 115}{2}\right) = 102.5$$

So, the after-tax profit with hedging (point B on the graph) is:

$\frac{1}{2}(102.5 - 100) = 1.25$, *ie* with hedging there is an expected net profit of 1.25.

Solution 12.3

	Long 106-put		Short 109-call		Hedged position	
Asset price	Payoff	Profit	Payoff	Profit	Payoff	Profit
104	2	–6.05274	0	3.83714	2	–2.2156
105	1	–7.05274	0	3.83714	1	–3.2156
106	0	–8.05274	0	3.83714	0	–4.2156
107	0	–8.05274	0	3.83714	0	–4.2156
108	0	–8.05274	0	3.83714	0	–4.2156
109	0	–8.05274	0	3.83714	0	–4.2156
110	0	–8.05274	–1	2.83714	–1	–5.2156
111	0	–8.05274	–2	1.83714	–2	–6.2156
112	0	–8.05274	–3	0.83714	–3	–7.2156



Note: the current asset price is not required for the naked position but would have been required to determine the payoff/profit profile of the hedged position (not asked for).

Solution 12.4

See Section 12.3 of the Session Notes.

Solution 12.5

1. Using a *long put* position, which combines with the long position in the underlying product to produce a long floor position.

2. Using a *collar* (long position in lower strike price put and short position in higher strike price call). Combined with the long position in the underlying, this produces a bull spread, which still provides some limited upside profit exposure (although the profit doesn't increase beyond the call strike price).

3. Using a *ratio hedge* (two long lower strike price puts plus a short higher strike price put), which will produce an increasing profit once the underlying price exceeds the higher strike price.

Solution 12.6

If the price of corn in one year turns out to be S_T, Farmer Brown's unhedged net income will be:

$$S_T - 175$$

Also, the profit from a put will be:

$$Max(0, K - S_T) - FV_T(P_0)$$

The following table shows the actual and expected profits from each put.

Put option	Actual profit if price = \$234	Actual profit if price = \$174	Expected profit
strike price = \$180	\$48.36	–\$5.64	\$21.36
strike price = \$195	\$42.43	\$3.43	\$22.93
strike price = \$210	\$34.99	\$10.99	\$22.99

For example, if the actual corn price is \$234, then the \$180 strike price put will produce a profit of:

$$\begin{aligned} & S_T - 175 + Max(0, K - S_T) - FV_T(P_0) \\ &= 234 - 175 + 0 - 10.23 \times 1.04 \\ &= 48.36 \end{aligned}$$

Whereas, if the actual corn price is \$174, the \$180 strike price put will produce a profit of:

$$\begin{aligned} & S_T - 175 + Max(0, K - S_T) - FV_T(P_0) \\ &= 174 - 175 + (180 - 174) - 10.23 \times 1.04 \\ &= -5.64 \end{aligned}$$

The corresponding expected profit is then found by averaging these two values.

The table shows us that the \$210 strike price put will produce the highest expected profit.

