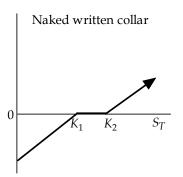


Solutions to practice questions – Study Session 11

Solution 11.1

A mandatory convertible bond has a payoff structure that resembles a written collar, so its price can be determined as a written collar.

The payoff to a naked written collar is shown below.



We saw that a naked written collar consists of selling a lower strike put and buying a higher price call. The same payoff graph can also be achieved by buying the underlying stock, selling a lower strike call, and buying a higher strike call.

You should be able to verify that the above payoff diagram can be achieved by either of the two strategies.

Name	Position	Equivalent position		
Floor or insured asset	Long asset + purchased put	Long bond + Purchased call		
Covered written call	Long asset + written call	Long bond + written put		
Cap or insured short	Short asset + purchased call	Short bond + purchased put		
Covered written put	Short asset + written put	Short bond + written call		

Solution 11.2

Solution 11.3

Expectation						
	Volatility					
Prices	Increase	Neutral view	Decrease			
Increase	Purchase call	Purchase asset	Sell or write put			
Neutral	Purchase straddle	No action	Sell or write straddle			
Decrease	Purchase put	Sell asset	Sell or write call			

The graphs are contained in your Study Session 11 notes!

Solution 11.5

The correct answer is A.

Both the assertion and the reason are true, and the reason is the correct explanation. If the asset price increases, the futures price for the asset also increases, resulting in a profit from the futures position that offsets the increase in price of the asset.

Solution 11.6

The correct answer is A.

Solution 11.7

The correct answer is A.

Both the assertion and reason are true. The reason explains the assertion. Since the only time to exercise an American call early is just before a dividend payment, it follows that a call option on a stock that does not make dividend payments should not be exercised before maturity.

Solution 11.8

The correct answer is D.

Y relates to I and III

Solution 11.9

Buy 1010-strike call for : \$93.69	
Sell 1050-strike call for :	\$73.00
Sell 1010-strike put for :	\$41.88
Buy 1050-strike put for:	<u>\$60.02</u>
Cost is therefore:	\$38.83

		Example payoffs			
	Price of asset underlying options:	1000	1020	1040	1060
Long strangle	Buy call, strike 1010	0	10	30	50
	Buy put, strike 1050	50	30	10	0
Short strangle	Sell put, strike 1010	-10	0	0	0
	Sell call, strike 1050	0	0	0	-10
	Totals:	40	40	40	40

So, the guaranteed payoff from the straddle will be 40.

The implied interest rate over 4 months is therefore 3% as:

$$\frac{40}{38.83} = 1.03$$

Solution 11.10

Put call parity gives us that $C_0 = 30 + 4.14 - \frac{30}{1.06^{0.5}} = 5.00$

A *floor* is a combination of a long stock position and a long put position on the same stock.

Let:

- S_0 and S_T be the stock price now and at the expiration date of the put option respectively
- P_0 be the current put option price
- *K* be the strike price of the put
- $FV(S_0)$ and $FV(P_0)$ be the future value of the initial stock price and put premium respectively.

The profit from the long stock position is given by:

 $S_T - FV(S_0)$

The profit from the long put position is given by:

 $Max(0, K - S_T) - FV_T(P_0)$

(i) Maximum loss

The floor will make a loss if the stock price falls below the strike price and the put is exercised.

The profit from the put will then be:

 $K - S_T - FV_T(P_0)$

So, the overall profit from the floor will be:

 $+S_T - FV(S_0) + K - S_T - FV_T(P_0) = -FV(S_0) + K - FV_T(P_0)$

ie the *loss* will be equal to the future value of the initial stock price, minus the strike price, plus the future value of the put premium.

This loss will be the same regardless of how far the stock price falls below the stock price and so represents the maximum loss from the floor.

(ii) Breakeven price

The floor will breakeven when the put is out-of-the-money, so that the profit on the stock exactly offsets the loss on the put.

The profit from the long put position will then be:

 $0 - FV_T(P_0)$

ie a loss equal to the future value of the put premium.

So, the overall profit from the floor will be:

 $+S_T - FV(S_0) + 0 - FV_T(P_0) = +S_T - FV(S_0) - FV_T(P_0)$

This will be equal to zero when:

 $+S_T = FV(S_0) + FV_T(P_0)$

ie the breakeven price is the future value of the initial stock price, plus the future value of the put premium.

A *covered call* is the combination of a long stock position and a short call position on the same stock. Let:

- S_0 and S_T be the stock price now and at the expiration date of the put option respectively
- *C*₀ be the current call option price
- *K* be the strike price of the call
- $FV(S_0)$ and $FV(C_0)$ be the future value of the initial stock price and call premium respectively.

The profit from the long stock position is given by:

 $S_T - FV(S_0)$

The profit from the short call position is given by:

 $-\{Max(0,S_T-K)-FV_T(C_0)\}$

(i) Maximum profit

The covered call will make a profit if the stock price rises.

If the stock price at expiration exceeds the strike price of the call option, then the option will be exercised, giving an overall profit to the covered call holder of:

$$+S_T - FV(S_0) - \{S_T - K - FV_T(C_0)\} = K + FV(C_0) - FV(S_0)$$

ie the maximum profit is the strike price, plus the future value of the call price, minus the future value of the initial stock price.

This will be the same regardless of the extent by which the stock price exceeds the strike price and so represents the maximum profit to the covered call position.

(ii) Maximum loss

The covered call will make a loss if the stock price falls. In fact, the lower the stock price at expiration, the greater loss. The maximum loss will therefore occur if the stock price falls to zero.

The profit from the call will then be:

 $-\{0-FV_T(C_0)\}$

So, the overall profit from the covered call will be:

$$+0 - FV(S_0) - 0 + FV_T(C_0) = +FV(C_0) - FV_T(S_0)$$

ie the *loss* will be equal to the future value of the initial stock price minus the future value of the call price.

A synthetic long forward consists of a combination of a long call and a short put position.

The profit from the long call is given by:

 $Max(0, S_T - K) - FV_T(C_0) = Max(0, S_T - 11.00) - 0.80 \times 1.05$

The profit from the short put is given by:

 $-Max(0, K - S_T) + FV_T(P_0) = -Max(0, 11.00 - S_T) + 1.65 \times 1.05$

So, if the stock price at expiration is equal to the strike price of \$11, the overall profit from then synthetic forward will be:

 $= 0 - 0.80 \times 1.05 - 0 + 1.65 \times 1.05$ = +0.8925

The breakeven price must therefore be less than the strike price of \$11.

If this is the case, the overall profit from then synthetic forward will be:

 $= 0 - 0.80 \times 1.05 - (11.00 - S_T) + 1.65 \times 1.05$ = S_T - 10.1075

ie the breakeven price is \$10.1075.

Solution 11.14

A purchased naked collar position involves buying lower strike price puts and buying higher strike price calls.

The profit from each put purchased will be:

 $+Max(0, K_1 - S_T) - FV_T(P_0^{K_1})$

Whilst the profit from each call sold will be:

$$-Max(0, S_T - K_2) + FV_T(C_0^{K_2})$$

(i) $S_T = 10$

If the stock price at expiration is \$10, the overall profit each purchased collar will be:

$$+K_1 - S_T - FV_T (P_0^{K_1}) - 0 + FV (C_0^{K_2})$$

= 11 - 10 - 1.60 × 1.05 + 1.76 × 1.05
= 1.168

So, the profit from 1,000 such positions will be \$1,168.

(ii) *Maximum profit*

A purchased naked collar is a bearish strategy, as the profit increases as the stock price falls. The maximum profit will therefore be obtained if the stock price falls to zero. In this case, the profit from each collar position will be:

$$+K_1 - S_T - FV_T(P_0^{K_1}) - 0 + FV(C_0^{K_2})$$

= 11 - 0 - 1.60 \times 1.05 + 1.76 \times 1.05
= 11.168

Giving a maximum total profit of \$11,168.

An *asymmetric butterfly spread* involves buying the lowest and highest price calls and selling the call with the middle strike price.

If we denote the three strike prices here by:

 $K_1 = 22$, $K_2 = 25$ and $K_3 = 30$

then the investor will need to buy and sell the calls in the following combination:

- buy λ \$22 calls
- sell 1 \$25 call
- buy 1λ \$30 calls

where:

$$\lambda = \frac{K_3 - K_2}{K_3 - K_1} = \frac{30 - 25}{30 - 22} = \frac{5}{8}$$

So, given that the investor buys 125 of the \$22 call options, she will also need to:

- sell 200 \$25 calls
- buy 75 \$30 calls.

The net cost of these options will be:

 $125 \times 5.32 - 200 \times 3.91 + 75 \times 2.28 = 54

Her overall profit at expiration will be given by:

$$+125[Max(0, S_T - 22) - 5.32 \times 1.05]$$

-200[Max(0, S_T - 25) - 3.91 \times 1.05]
+75[Max(0, S_T - 30) - 2.28 \times 1.05]

So, if $S_T = 25$, her profit will be:

 $+125[(25-22)-5.32\times1.05]-200[(0)-3.91\times1.05]+75[(0)-2.28\times1.05]$ = 318.30

which is, of course, her maximum profit from the trade.

Solution 11.16

The correct answer is D. A butterfly spread is a nondirectional trade.

Financial Mathematics

Solution 11.17

The correct answer is C.

Recall that the profit from a long call is given by:

 $Max(0, S_T - K) - FV_T(C_0)$

So, the combined profit from the three calls making up the butterfly spread is given by:

 $+Max(0, S_T - 60) - 8.25 \times 1.04 - 2Max(0, S_T - 65) + 2 \times 6.14 \times 1.04 + Max(0, S_T - 70) - 4.50 \times 1.04$

The maximum loss occurs when either all three options are exercised, *ie* when $S_T > 70$, or when none of the options is exercised, *ie* when $S_T < 60$.

If $S_T > 70$, the profit is:

 $+(S_T - 60) - 8.25 \times 1.04 - 2(S_T - 65) + 2 \times 6.14 \times 1.04 + (S_T - 70) - 4.50 \times 1.04$ = (-8.25 + 2 × 6.14 - 4.50) × 1.04 = -0.49

If $S_T < 60$, the profit is:

 $\begin{aligned} +(0) - 8.25 \times 1.04 - 2(0) + 2 \times 6.14 \times 1.04 + (0) - 4.50 \times 1.04 \\ = (-8.25 + 2 \times 6.14 - 4.50) \times 1.04 \\ = -0.49 \end{aligned}$

So, the maximum loss is \$0.49

Note that the loss is equal to the accumulated value of the net initial debit.

Solution 11.18

Recall that the profit from the long put position is given by:

$$Max(0, K-S_T)-FV_T(P_0)$$

Also, the profit from a long call is given by:

 $Max(0, S_T - K) - FV_T(C_0)$

So, the overall profit from the trade is:

 $+Max(0,60-S_T)-5.90\times1.04+2Max(0,S_T-60)-2\times8.25\times1.04$

If $S_T > 60$, the profit is:

 $+(0) - 5.90 \times 1.04 + 2(S_T - 60) - 2 \times 8.25 \times 1.04$ $= 2S_T - 143.296$

So, she will therefore make a profit if:

 $2S_T > 143.296$

ie $S_T > 71.65

If $S_T < 60$, the profit is:

 $+(60 - S_T) - 5.90 \times 1.04 + 2(0) - 2 \times 8.25 \times 1.04$ = 36.704 - S_T So, she will make a profit if:

 $S_T < 36.70

Notice that her profit increases the further S_T is above \$71.65 and the further S_T is below \$36.70. She therefore expects the stock price to move considerably away from its current value of \$60, although she may be unsure in which direction it will move.

Actually, she may believe that it is a bit more likely to move considerably upwards. This is because she has purchased two calls and will therefore receive \$2 of additional profit for each extra dollar by which the stock price exceeds \$65.51 (whereas she makes only \$1 additional profit for each extra dollar the stock price falls below \$48.98).

This combined trade is known as a strap. A combination of two puts and one call with the same strike price on the same underlying asset is known as a strip.