SOA Exam C
CAS Exam 4
Flashcards

Spring 2009 exams

Key concepts

Important formulas

Efficient methods

Advice on exam technique

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HOW TO USE THESE FLASHCARDS

These flashcards are designed to help you to prepare efficiently in the run-up to the Course C exam of the Society of Actuaries. They include conceptual ideas, key formulas and techniques for efficient problem solving. The Course C Exam has a number of problems that require first principles reasoning as well as a fair amount of computation. So don't look at the lists of formulas as simply being memorization work. There are often simple intuitive ideas that underlie the formulas as well as basic mathematical reasons why they are correct. Strive to understand and learn the key relations from these points of view and your knowledge will not be the superficial type that may collapse under the stress of taking the exam. The more that you understand, the easier it becomes to retain the key ideas and write them down quickly and accurately.

We have designed the flashcards so that they can be carried conveniently and read frequently in the final run-up to the exam, *eg* when commuting to work. We hope that you will personalize them by adding your own comments and notes, and checking each section when you feel confident with the material covered.

You will probably also find these summaries useful when you are at the stage of working through the past exams. If you see a particular point being examined that is not summarized here add it to these flashcards. Let us know if you find some key ideas that are missing.

Good luck with your studying.

Moment generating function

$$M_X(t)=E\left[e^{tX}\right]=\sum_x e^{tx}\Pr(X=x) \qquad \text{(discrete case)}$$

$$=\int_{-\infty}^{\infty} e^{tx} f_X(x)\,dx \qquad \text{(continuous case)}$$

Properties of moment generating functions

1. $E\left[X^k\right]=M_X^{(k)}(0)$ provided that the derivative exists.

2. If $M_X(t)=M_Y(t)$, then X and Y are identically distributed.

3. If $X_1, X_2, \dots, X_n$ are independent random variables, then:

$$M_{\sum a_i X_i}(t)=M_{X_1}(a_1 t)\cdots M_{X_n}(a_n t)$$

Probability generating function

$$P_X(t)=E\left[t^X\right]=E\left[e^{\ln(t)X}\right]=M_X(\ln(t))$$

Properties

1. $P_X^{(k)}(1)=E\left[X(X-1)\cdots(X-k+1)\right]$ for $k=1,2,\dots$ if the derivative exists.

2. If $P_X(t)=P_Y(t)$, then X and Y are identically distributed.

3. If $X_1, X_2, \dots, X_n$ are independent random variables, then:

$$P_{\sum X_i}(t)=P_{X_1}(t)\cdots P_{X_n}(t)$$

4. $P_X(t)=M_X(\ln(t)) \iff P_X\left(e^t\right)=M_X(t)$

Exact distribution of a sum of independent variables

If $f_X(x)$ and $f_Y(y)$ are the probability functions or PDF's for independent random variables X and Y, then the probability function or PDF for the sum $S=X+Y$ is referred to as the *convolution of* f_X *with* f_Y and is denoted $f_X * f_Y(s)$:

$$f_X * f_Y(s) = \sum_{0 \le x \le s} f_X(x) f_Y(s-x) \quad \text{(discrete non-negative)}$$

$$f_X * f_Y(s) = \int_0^s f_X(x) f_Y(s-x)\,dx \quad \text{(continuous non-negative)}$$

If $X_1, X_2, \ldots, X_n$ are independent and identically distributed like X, then the PDF of the sum $S=X_1+\cdots+X_n$, denoted by $f_X^{*n}(s)$, is computed recursively by the rule:

$$\begin{aligned} f_X^{*k+1}(s) &= f_X^{*k} * f_X(s) \qquad k \ge 1 \\ &= \sum_{0 \le x \le s} f_X^{*k}(x) f_X(s-x) \end{aligned}$$

where $f_X^{*1}(x)=f_X(x)$ and $f_X^{*0}(0)=1$.

The individual risk model (IRM)

Suppose that a portfolio consists of n risks (policies) whose *total annual losses* $X_1, X_2, \ldots, X_n$ are independent and identically distributed like X. The aggregate annual loss for the entire portfolio is represented by the sum:

$$S=X_1+X_2+\cdots+X_n$$

Apply the Central Limit Theorem and approximate the distribution of S by a normal distribution with mean $\mu=nE[X]$ and variance $\sigma^2=n\,\text{var}(X)$.

IRM properties

1. $E[S] = n\,E[X]$, $\text{var}(S) = n\,\text{var}(X)$

2. If $n \geq 50$, then:

$$\Pr(S \leq F) = \Pr\left(\frac{S - E[S]}{\sqrt{\text{var}(S)}} \leq \frac{F - E[S]}{\sqrt{\text{var}(S)}}\right)$$

$$\approx \Phi\left(\frac{F - E[S]}{\sqrt{\text{var}(S)}}\right) = \Phi\left(\frac{F - nE[X]}{\sqrt{n\,\text{var}(X)}}\right)$$

where $\Phi(x)$ is the cumulative distribution function of the standard normal distribution.

3. The approximate $100(1-\alpha)$ th percentile of S is:

$$E[S] + z_\alpha \sqrt{\text{var}(S)} = n\,E[X] + z_\alpha \sqrt{n\,\text{var}(X)}$$

where $\alpha = \Pr(N(0,1) > z_\alpha) = 1 - \Phi(z_\alpha)$.

Double expectation theorem

For any random variables X and Y where X may depend on Y, we have:

$$E[X] = E\big[E[X|Y]\big]$$
$$\text{var}(X) = E\big[\text{var}(X|Y)\big] + \text{var}\big(E[X \mid Y]\big)$$

One essential use is to develop moment formulas for the collective risk model:

$S = X_1 + \cdots + X_N$ where the X_i are independent and identically distributed like X, and N is independent of the X_i

If the number of terms is given, we can compute the mean and variance from $E[S|N=n] = nE[X]$ and $\text{var}(S|N=n) = n\,\text{var}(X)$. Write these relations as $E[S|N] = N\,E[X]$, $\text{var}(S|N) = N\,\text{var}(X)$.